On Equivalence of
Automated Market Maker and Limit Order Book Systems

Jamie E. Young*

JEY Labs

October 2020

Abstract

We prove that in a two token economy, the state of an Automated Market Maker (AMM) is mathematically equivalent to a Limit Order Book (LOB). Specifically, we show that for any AMM with a particular state, there is a corresponding LOB. Conversely, for any LOB, there is a corresponding AMM with a particular state, under certain regularity conditions. Accordingly, we explore AMM and LOB as interchangeable approaches to market systems design. We notice that crucially LOB allows an expressive maker strategy space to support liquidity, while AMM sacrifices degrees of freedom on market maker strategies to prioritize transparency, inclusiveness, and fairness. In particular we highlight the potential for hybrid AMM and LOB designs, and provide a few directions of development. Current LOB systems can disintermediate liquidity reserves from the strategies that require them, thus tailoring towards more transparency, inclusiveness, and fairness. Meanwhile, AMM systems can be tailored for finer grained market mechanisms (e.g. an AMM that mirrors LOB liquidity).

1 Preface

In this paper, we focus on showing that the Automated Market Maker (AMM) approach to market design creates market states that are equivalent to corresponding Limit Order Book (LOB) microstructure. At its core, our equivalence proof serves as a basis for further discussion on the AMM as a viable alternative to LOB designs for market systems.

The paper is structured as follows, we first mathematically define AMM and order book and their defining properties, respectively. We show AMM state and order book equivalence holds for a two token economy; for any arbitrary AMM state its implied order book counterpart can be deduced, and vice versa. We then illustrate that the AMM state and order book equivalence holds for Uniswap's AMM as an example case. Finally we end with a discussion of AMM and LOB, their benefits, drawbacks and open design questions, and look towards the possibility of hybrid designs which combine the benefits of both.

2 Mathematical Formulation

We proceed to define AMM and LOB formally. For ease of exposition, we assume a two token economy supporting swap trades between the two tokens. We define \$X as the base token and \$Y as the quote token. Price is denoted as the exchange rate for one unit of \$X expressed in units of \$Y.

*ENS: professorjey.eth. Contact: @ProfessorJEY.

2.1 AMM

We define a *general* AMM liquidity pool as {\$X, \$Y} with a current balance: x_0 of \$X, and current balance: y_0 of \$Y. Let this AMM liquidity pool take mathematical representation as an object consisting of the balance: x of \$X, and the balance: y of \$Y, that satisfies a certain invariance function $I(x, y) = 0$ for $x \geq 0, y \geq 0$, at a particular *state* (x_0, y_0), with the following properties:

1. $y = f(x)$ is a *semi-differentiable* function as implied by $I(x, y) = 0$, with $x \geq 0$ and $y \geq 0$, and $y_0 = f(x_0)$
2. $f'_-(x) \leq 0$ and $f'_+(x) \leq 0$ for all $x \in \mathcal{D}(f)^o$, where $\mathcal{D}(f)^o$ denotes the interior of the domain of f
3. For any x_1, x_2 in $\mathcal{D}(f)^o$ such that $x_1 \leq x_2$, we have $f'_-(x_1) \leq f'_+(x_1) \leq f'_-(x_2) \leq f'_+(x_2)$

The general AMM definition above is highly technical and hard to work with. We now define a *smooth* AMM with the following properties:

1. $y = f(x)$ is a differentiable function, with domain $\mathcal{D}(f) = (0, \infty)$, and $y_0 = f(x_0)$
2. f' is continuous and $f'(x) < 0$ for all $x \in \mathcal{D}(f)$
3. $f'(x)$ is strictly increasing for all $x \in \mathcal{D}(f)$
4. $\lim_{x \to 0} f'(x) = -\infty$
5. $\lim_{x \to \infty} f(x) = 0$

It is clear that a smooth AMM satisfies all the requirements of a general AMM. In this paper we will focus on the analysis of smooth AMM.

The function f is the *trading function* of the AMM: from the perspective of the liquidity pool, if its initial balance is x_0 of \$X and y_0 of \$Y, and if its balance of \$X changes by Δx, then its balance of \$Y changes by $\Delta y = f(x_0 + \Delta x) - y_0$. By Property 2, f is monotonically decreasing, it can be proved that when $\Delta x > 0$, we have $\Delta y < 0$, and when $\Delta x < 0$, we have $\Delta y > 0$. This implies that if a trader swaps $\Delta x > 0$ of \$X, the amount of \$Y they take out is $-\Delta y > 0$. Similarly, if a trader wants to take out some \$X from the pool, in this case $\Delta x < 0$, then $\Delta y > 0$, which is the amount of \$Y that the trader needs to swap into the pool.

At any state (x_0, y_0) of the AMM, the instantaneous spot price (or marginal price) of \$X in terms of \$Y is $-f'(x_0)$. To see this, recall that the economic meaning of the price of \$X in \$Y is just the amount of \$Y the trader needs to swap in exchange for per unit of \$X, or equivalently the amount of \$Y one receives in exchange for swapping 1 unit of \$X. From the above analysis, this equals to $-\Delta y/\Delta x$. Taking the limit, we have $\lim_{x \to x_0} -\Delta y/\Delta x = \lim_{x \to x_0} -dy/dx = -f'(x_0)$. By Property 2, this is guaranteed to be positive, which means that price is always positive at any state of the AMM.

Property 3 says that the marginal price as a function $p(x) = -f'(x)$ of x is decreasing. This guarantees that as the balance of \$X in the pool increases (which means traders sell to the pool), the price is decreasing. This is an essential property to ensure stability of the system, because if traders swap \$X to the pool and the price keeps increasing, this will incentivize traders to swap even more \$X to the pool, causing instability. Property 3 also implies that if $f''(x)$ exists, it has to be positive.

By enforcing $0 \in \overline{\mathcal{D}(f)}$ and $0 \in \overline{\mathcal{R}(f)}$, it follows that the smooth AMM should use all of its \$X and \$Y reserves to provide liquidity. Although technically an AMM is not required to use up all of its reserve to provide liquidity, this would be *isomorphic* to another AMM with the same pricing curve that uses up all the reserve to provide liquidity. So we will only consider AMMs that satisfy this property without loss of generality.

Note that the definition of smooth AMM implies that $\lim_{x \to \infty} f'(x) = 0$.

2.2 Order Book

An Order Book in a trading pair \$X-\$Y can be mathematically represented as a function $q(p)$ where p is the price of \$X in \$Y, and for $p < p_0$, $q(p)$ represents the cumulative limit bid quantity (in units of \$X) from p_0 to p, for $p > p_0$, $q(p)$ represents the cumulative limit ask quantity (in units of \$Y) from p_0 to p, where p_0 is mid order book price. The function $q(p)$ needs to satisfy the following properties:

1. $p \geq 0$ and $q(p) \geq 0$
2. $q(p_0) = 0$ where p_0 is the mid order book price
3. $q(p)$ is non-increasing for all $p < p_0$ and $q(p)$ is non-decreasing for all $p > p_0$

We will also define a *smooth* order book with better regularity conditions as follows:

1. $q(p)$ is continuous with domain $\mathcal{D}(q) = (0, \infty)$ and range $\mathcal{R}(q) = [0, \infty)$
2. $q(p_0) = 0$ where p_0 is the mid order book price
3. $q(p)$ is strictly decreasing for all $p < p_0$ and $q(p)$ is strictly increasing for all $p > p_0$
4. $\lim_{p \to \infty} q(p)$ is finite
5. $\int_0^{p_0} q(p)dp$ is finite

It is clear that a smooth order book satisfies all the requirements of a general order book. In this paper we will focus on the analysis of smooth order book.

Property 1 implies that price can only approach but never actually reach 0. Also, the cumulative bid quantity in units of \$X is not finite.

Property 2 states that by definition, there are no bid or ask quantities at the mid price.

Property 3 essentially establishes that as price moves away from the mid price on the bid side, the cumulative bid quantity increases, and as price moves away from mid price on the ask side, the cumulative ask quantity increases as well.

Property 4 states that the total limit sell quantity needs to be finite, which should usually be satisfied because the total circulating supply of \$X should be finite.

Property 5 states that the total bid quantity in units of \$Y needs to be finite, which should usually be satisfied because the total circulating supply of \$Y should be finite. (Note that this does not contradict with Property 1, which says the total bid quantity in units of \$X is not finite.).

Practical Approximation of $q(p)$: In the practice, an order book will never be continuous, because it always has discrete price levels, making $q(p)$ a step function. Also, it is not strictly monotonic. However, if the price levels are granular enough, the actual function could be well approximated by a continuous function with the desired strict monotonicity property.

3 Equivalence

3.1 Implied order book by AMM

Given a smooth AMM $y = f(x)$ at a particular state (x_0, y_0) satisfying $y_0 = f(x_0)$ where x_0 is the current balance of \$X and y_0 is the current balance of \$Y, we can deduce the smooth order book implied by this smooth AMM.

We have $p = -f'(x)$ where p is the price of \$X in \$Y as a function of x, and $p_0 = -f'(x_0)$ is the current marginal price. Therefore, $x = f'^{-1}(-p)$ represents the pool balance of \$X at price p. Since we have made the assumption that f' is strictly increasing, $x = f'^{-1}(-p)$ is well defined. For the case $p < p_0$, due to the monotinicity of f', we have $x > x_0$. This is saying that if a trader comes to the pool and sells $X up to the price p, he would be able to sell a total quantity of $(x - x_0)$ in $X. This is equivalent to saying that the cumulative limit bid quantity from price p_0 up to p as implied by the AMM is $(x - x_0) = f'^{-1}(-p) - x_0$. On the other hand, for the case $p > p_0$, we have $x < x_0$. This is saying that if a trader comes to the pool and buys $X up to the price p, he would be able to buy a total quantity of $(x_0 - x)$ in $X. This is equivalent to saying that the cumulative limit ask quantity from price p_0 up to p as implied by the AMM is $(x_0 - x) = x_0 - f'^{-1}(-p)$. Therefore, we have deduced the following function $q(p)$:

$$q(p) = \begin{cases} f'^{-1}(-p) - x_0 & \text{if } p < p_0 \\ 0 & \text{if } p = p_0 \\ x_0 - f'^{-1}(-p) & \text{if } p > p_0 \end{cases}$$

We will verify that the function $q(p)$ represents a smooth order book.

First, Property 2 is verified by definition.

Since f' is strictly increasing, f'^{-1} is well defined and also strictly increasing, hence $f'^{-1}(-p)$ is decreasing in p. Therefore $q(p)$ is decreasing for $p < p_0$ and increasing for $p > p_0$. So Property 3 is satisfied.

Since f' is continuous, f'^{-1} is also continuous. Therefore q is continuous for $p < p_0$ and for $p > p_0$. Since $p_0 = -f'(x_0)$, we have $x_0 = f^{-1}(-p_0)$, from which we can verify that $q(p)$ is also continuous at p_0, hence $q(p)$ is everywhere continuous. Since $\mathcal{D}(f'^{-1}) = \mathcal{R}(f') = (-\infty, 0)$, we have $\mathcal{D}(q) = (0, \infty)$. Since $\mathcal{R}(f'^{-1}) = \mathcal{D}(f) = (0, \infty)$, we can show that $\mathcal{R}(q) = [0, \infty)$. So Property 1 is satisfied.

For Property 4, since $\lim_{x \to 0} f'(x) = -\infty$, we have $\lim_{p \to \infty} f'^{-1}(-p) = 0$, therefore $\lim_{p \to \infty} q(p) = \lim_{p \to \infty} x_0 - f'^{-1}(-p) = x_0$ which is the pool $X balance and is finite.

For Property 5, note that the integral represents the area under the $q(p)$ curve from $p \in (0, p_0)$, which can be reparameterized by viewing as the area under the curve of $p(q)$ from $q \in (0, +\infty)$ where $p(q)$ is the inverse function of $q(p)$. Since for $p < p_0$, we have $p(q) = -f'(q + x_0)$, therefore $\int_0^{p_0} q(p)dp = \int_0^{\infty} p(q)dq = \int_0^{\infty} -f'(q + x_0)dq = \int_{x_0}^{\infty} -f'(t)dt = f(x_0) = y_0$ since $\lim_{x \to \infty} f(x) = 0$. Therefore $\int_0^{p_0} q(p)dp = y_0$ is finite, and in fact equal to the pool $Y balance, satisfying Property 5.

We have shown that the function $q(p)$ defined above indeed represents a smooth order book, with $p_0 = -f'(x_0)$ as the mid price.

3.2 Implied AMM by order book

Given a smooth order book $q(p)$, we can deduce the smooth AMM at a particular state implied by this order book.

We will work backwards to deduce the AMM trading function. First, given $q(p)$, define the following:

$$B(p) = \begin{cases} q(p) & \text{if } p < p_0 \\ 0 & \text{if } p = p_0 \\ -q(p) & \text{if } p > p_0 \end{cases}$$

$$x_0 = \lim_{p \to \infty} q(p) = \lim_{p \to \infty} -B(p)$$
$$y_0 = \int_0^{p_0} q(p)dp = \int_0^{p_0} B(p)dp > 0$$

Note that x_0 is guaranteed to exist by Property 4 and y_0 is guaranteed to exist by Property 5. Since $q(p)$ is continuous, $B(p)$ is also continuous, and by Property 3, $B(p)$ is strictly decreasing for all p. By construction, $\mathcal{D}(B) = (0, \infty)$ and $\mathcal{R}(B) = (-x_0, \infty)$. Note that since $B(.)$ is monotonically decreasing, $B^{-1}(.)$ is well defined, with $\mathcal{D}(B^{-1}) = \mathcal{R}(B) = (-x_0, \infty)$, $\mathcal{R}(B^{-1}) = \mathcal{D}(B) = (0, \infty)$. Since $B(.)$ is continuous, $B^{-1}(.)$ is also continuous.

As motivated by the form of $q(p)$ as in section 3.1, we would like to solve for a function f that satisfies $B(p) = f'^{-1}(-p) - x_0$, Rearranging, we have $B(p) + x_0 = f'^{-1}(-p) \Rightarrow f'(B(p)+x_0) = -p$. Let $x = B(p) + x_0 \Rightarrow p = B^{-1}(x - x_0)$. Plugging in, we get $f'(x) = -B^{-1}(x - x_0)$. Setting $f(x_0) = y_0$ and solve for $f(x)$, we get $f(x) = y_0 + \int_{x_0}^{x} -B^{-1}(t - x_0)dt = y_0 - \int_0^{x-x_0} B^{-1}(t)dt$.

To prove that $f(.)$ indeed represents a smooth AMM, we would need to verify that it satisfies all the required properties.

Clearly f is differentiable since we started with f' and integrate to get f. $\mathcal{D}(f) = \mathcal{D}(f') = \{x : x - x_0 \in \mathcal{D}(B^{-1})\} = (0, \infty)$. Also by construction, $y_0 = f(x_0)$, so Property 1 is satisfied. Since B^{-1} is everywhere continuous, f' is everywhere continuous, and since $B^{-1}(.)$ is monotonically decreasing, $f'(.)$ is monotonically increasing. Since $\mathcal{R}(B^{-1}) = (0, \infty)$, $\mathcal{R}(f') = (-\infty, 0)$. Therefore Property 2, Property 3 and Property 4 of smooth AMM are verified.

For Property 5,we have $\lim\limits_{x\to\infty} f(x) = \lim\limits_{x\to\infty} y_0 - \int_0^{x-x_0} B^{-1}(t)dt = y_0 - \lim\limits_{x\to\infty} \int_0^{x} B^{-1}(t)dt = y_0 - \int_0^{\infty} B^{-1}(t)dt$. Change variable by $p = B^{-1}(t)$, then $t = B(p)$, and as t ranges over $(0, \infty)$, p ranges over $(p_0, 0)$. Therefore the integral can be rewritten as $\int_0^{\infty} B^{-1}(t)dt = \int_{p_0}^{0} p dB(p) = pB(p)\Big|_{p=p0}^{p=0} - \int_{p_0}^{0} B(p)dp = \int_0^{p_0} B(p)dp = y_0$. Therefore $\lim\limits_{x\to\infty} f(x) = 0$, satisfying Property 5.

We have shown that the function $f(x)$ and the tuple (x_0, y_0) defined above together indeed represent a smooth AMM at state (x_0, y_0).

4 Example

We present a verification that AMM State and Order Book equivalence holds for Uniswap's AMM as an example.

4.1 Uniswap

Uniswap offers a two token swap for over 8,000 pairs and counting. It is implemented in a series of open Ethereum smart contracts, consequently its AMM internal mechanism is publicly known to participants [7].

The invariant of an Uniswap pool is $xy = k$, which implies a trading function $y = f(x) = k/x$. One can verify that this is indeed a smooth AMM. At a particular state (x_0, y_0) satisfying $y_0 = f(x_0)$, the marginal price is $p_0 = -f'(x_0) = k/x_0^2$. Therefore $k = p_0 x_0^2$, and $f(x) = p_0 x_0^2/x$, $f'(x) = -p_0 x_0^2/x^2$, $p = -f'(x) = p_0 x_0^2/x^2$. So $f'^{-1}(-p) = x_0\sqrt{p_0/p}$. According to the formula, the corresponding smooth order book function is:

$$q(p) = \begin{cases} x_0(\sqrt{p_0/p} - 1) & \text{if } p < p_0 \\ 0 & \text{if } p = p_0 \\ x_0(1 - \sqrt{p_0/p}) & \text{if } p > p_0 \end{cases}$$

where $p_0 = -f'(x_0)$ is the current mid price and x_0 is the current \$X balance of the pool.

Conversely, if we have an order book with

$$q(p) = \begin{cases} x_0'(\sqrt{p_0/p} - 1) & \text{if } p < p_0 \\ 0 & \text{if } p = p_0 \\ x_0'(1 - \sqrt{p_0/p}) & \text{if } p > p_0 \end{cases}$$

where p_0 is the mid price and x_0' is a parameter, one can verify that this is indeed a smooth order book. Then we have $B(p) = x_0'(\sqrt{p_0/p} - 1)$, $x_0 = \lim\limits_{p\to\infty} q(p) = x_0'$, $y_0 = \int_0^{p_0} q(p)dp = \int_0^{p_0} x_0'(\sqrt{p_0/p} - 1)dp = x_0'(2\sqrt{p_0}\sqrt{p} - p)\Big|_{p=0}^{p=p_0} = x_0' p_0$. Then $B^{-1}(x) = p_0/(x/x_0' + 1)^2$. Therefore $f(x) = y_0 + \int_{x_0}^{x} -B^{-1}(t - x_0)dt = \int_{x_0'}^{x} -B^{-1}(t - x_0')dt = y_0 + \int_{x_0'}^{x} -p_0/((t - x_0')/x_0' + 1)^2 dt = y_0 + \int_{x_0'}^{x} -p_0/((t/x_0')^2 dt = y_0 - \int_{x_0'}^{x} p_0 x_0'^2/t^2 dt =$

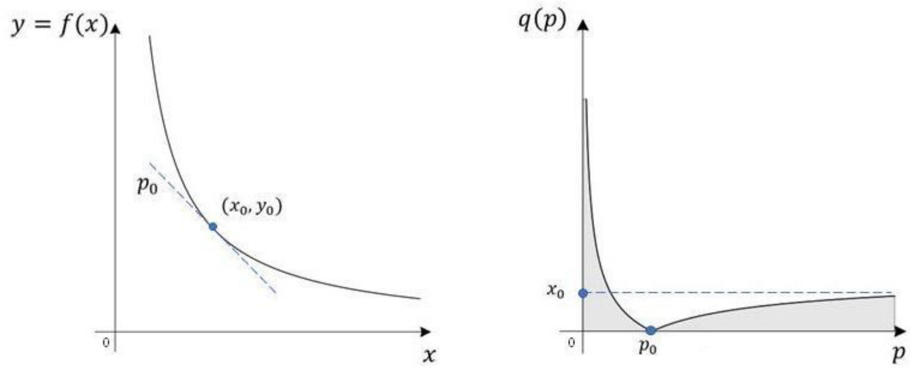

Figure 1: Uniswap AMM pricing curve with particular state(x_0,y_0) (left), and its corresponding order book (right).

$y_0 + p_0 x_0'^2 t^{-1}\Big|_{t=x_0'}^{t=x} = y_0 + p_0 x_0'^2(\frac{1}{x} - \frac{1}{x_0'}) = \frac{p_0 x_0'^2}{x} + y_0 - p_0 x_0' = \frac{p_0 x_0'^2}{x}$, which is indeed the Uniswap trading function, with the current state at $(x_0', x_0' p_0)$ and marginal price p_0.

4.2 Uniswap Features

Uniswap strikes a design extreme for being a broadly inclusive and transparent market, with two defining features. Firstly, market participants for both liquidity provisioning (makers) and takers face virtually no added barriers to participation beyond the same barriers for interacting on the underlying Ethereum blockchain. Secondly, the specification of an AMM invariant $xy = k$ removes degrees of freedom for expressing liquidity provisioning preferences, thus subjecting all liquidity providers at a given point to a homogeneous shared strategy.

5 Towards a Hybrid of AMM and LOB Systems

Our mathematical proof places us in ongoing discussion about the efficacy of AMM market systems. We provide a background briefing on LOB and then AMM, and then discuss documented benefits of each design approach, as well as their challenges. Overall, we notice that LOB crucially allows for a space of heterogeneous, expressive strategies to support its liquidity. Meanwhile, AMM prioritizes transparency, inclusiveness, and fairness but limits all participating liquidity (makers) to the same pre-declared pricing curve. This creates possibility for combining aspects of both design approaches for hybrid market systems.

5.1 Limit Order Book is Granular

For a two token market, an order book acts as a snapshot view, representing the reserve of available liquidity for the bid and ask side of the market. A LOB summarizes the available liquidity structure; at its finest level of granularity it is composed of open limit orders placed by committed market participants [8], with an implied ordering (e.g. by price then time) to distinguish precedence. The market operator incurs execution burden of *order matching* for any incoming participant *taker* order (crossing the market bid-ask spread) against available *maker* liquidity on the LOB. LOB market microstructure has its benefits, for instance, supporting long-term profitable *market maker* trading strategies [6, 11]. However LOB's come with inherent complexity and operational cost, because the market state is more granular to store, with resulting structural burden for maintenance and update after each market action.

5.2 AMMs in Decentralized Finance

Decentralized Finance (DeFi) for blockchain (e.g. Ethereum) has created interest in the design of AMM market systems implemented compactly as smart contracts on-chain with open access to participants (e.g. Uniswap [7], Balancer [9], Curve [4], etc). AMMs' expose to participants simpler liquidity provisioning en lieu of the richer limit order interface. Unlike the LOB design, under AMM, market maker participants cannot supply a conditional execution price parameter. They are constrained to simply adding and removing token liquidity based on the Automated Market Maker - a mathematical model for determining price for trades with liquidity pool supply (state) as its input. Recent work has showcased a growing diversity of AMM design by identifying families of mathematical models and their corresponding properties and challenges (e.g. susceptibility to front-running) [2, 1, 5, 12].

5.3 On Maker Preference, Openness, Inclusiveness and Fairness

Current AMM designs [7, 4, 9], simply do not keep information on each market maker's price preference, missing out on information otherwise available from keeping maker limit order prices in LOB systems. Thus, LOB is additionally complex and allows for market participants to express their true price preference, while keeping their diverse trading strategies private (model is hidden from other participants). By analogy, AMM can be likened to a LOB system which allows participation in its liquidity reserve with only one pre-declared, open, shared market making strategy. However, AMM offers broad participation and fairness, by maintaining homogeneous price preference across the liquidity reserve. Crucially the liquidity reserve is disintermediated from the content of the market maker strategy itself. AMM liquidity providers need only capital with no self-provided strategy needed, as the strategy is already deployed publicly for them. The AMM's openness comes at the cost of exposure of liquidity supply to arbitrage traders and front-runners, which is a well documented concern for a variety of decentralized exchanges on open blockchains [3]. However there is also evidence to indicate contextual or conditional stability of AMM designs may depend on market condition [2]. Assessing emergent properties, and any market performance gap between AMM and LOB remains an open topic, as aforementioned exposure of liquidity reserves, slippage, and impermanent loss (from underlying token price divergence) all factor into the market dynamics [10].

5.4 Hybrid AMM and LOB Systems

We recognize there is a path as well for hybrid designs. We envision at least a few possible approaches worth exploring:

1. LOB systems could seek more transparency and fairness by exposing more information granularity about liquidity (e.g. down to listing a limit order's owner publicly).
2. LOB systems could be more inclusive by taking AMM's lead in disintermediating liquidity capital contribution and a market making strategy.
3. AMM systems could incorporate mechanisms to continuously mirror the liquidity from LOB systems, to support more heterogeneity in true price preferences.

6 Conclusion

Our proof shows equivalence of AMM state and an order book. This equivalence proof serves as a basis for further discussion on enriching AMM and LOB designs. AMM designs can benefit from market maker preference, and LOB could adopt features to be more inclusive, transparent and fair. We anticipate that the strengths of both approaches can be brought together, not only towards addressing the current challenges of decentralized markets on blockchain, but also towards improving traditional financial markets.

References

[1] Guillermo Angeris and Tarun Chitra. Improved price oracles: Constant function market makers, 2020.

[2] Guillermo Angeris, Hsien-Tang Kao, Rei Chiang, Charlie Noyes, and Tarun Chitra. An analysis of uniswap markets, 2020.

[3] Philip Daian, Steven Goldfeder, Tyler Kell, Yunqi Li, Xueyuan Zhao, Iddo Bentov, Lorenz Breidenbach, and Ari Juels. Flash boys 2.0: Frontrunning, transaction reordering, and consensus instability in decentralized exchanges, 2019.

[4] Michael Egorov. Stableswap - efficient mechanism for stablecoin liquidity, 2019.

[5] Alex Evans. Liquidity provider returns in geometric mean markets, 2020.

[6] Puneet Handa and Robert A. Schwartz. Limit order trading. *The Journal of Finance*, 51(5):1835–1861, 1996.

[7] Noah Zinsmeister Hayden Adams and Dan Robinson. Uniswap v2 core, 2020.

[8] Paul Irvine, George Benston, and Eugene Kandel. Liquidity beyond the inside spread: Measuring and using information in the limit order book. *SSRN Electronic Journal*, 05 2000.

[9] F. Martinelli and N. Mushegian. A non-custodial portfolio manager, liquidity provider, and price sensor, 2019.

[10] Bancor Network. The v2 difference: How bancor's v2 update takes amms to the next level, 2020.

[11] Duane J. Seppi. Liquidity Provision with Limit Orders and a Strategic Specialist. *The Review of Financial Studies*, 10(1):103–150, 01 1997.

[12] Yongge Wang. Automated market makers for decentralized finance (defi), 2020.